Mobius Institute Board of Certification
Category IV Exam Reference Equations

Vibration unit conversions [English]

$$mils_{pk-pk} = \frac{19098\ in/s_{pk}}{f_{cpm}}$$

$$in/s_{pk} = \frac{5217\ G_{rms}}{f_{cpm}}$$

$$mils_{pk-pk} = \frac{9.958x10^7 G_{rms}}{f_{cpm}^2}$$

$$G_{rms} = \frac{f_{cpm}\ in/s_{pk}}{5217}$$

$$in/s_{pk} = \frac{f_{cpm}\ mils_{pk-pk}}{19098}$$

$$G_{rms} = \frac{f_{cpm}^2\ mils_{pk}}{9.958x10^7}$$

Vibration unit conversions [Metric]

$$\mu m_{pk-pk} = \frac{27009\ mm/s_{rms}}{f_{cpm}}$$

$$mm/s_{rms} = \frac{93712\ G_{rms}}{f_{cpm}}$$

$$\mu m_{pk-pk} = \frac{2.53x10^9 G_{rms}}{f_{cpm}^2}$$

$$G_{rms} = \frac{f_{cpm}\ mm/s_{rms}}{93712}$$

$$mm/s_{rms} = \frac{f_{cpm}\ \mu m_{pk-pk}}{27009}$$

$$G_{rms} = \frac{f_{cpm}^2\ \mu m_{pk-pk}}{2.53x10^9}$$

$$1\ G_{rms} = 9.81 m/s^2{}_{rms}$$

$$f_{cpm} = 60\ f_{hz}$$

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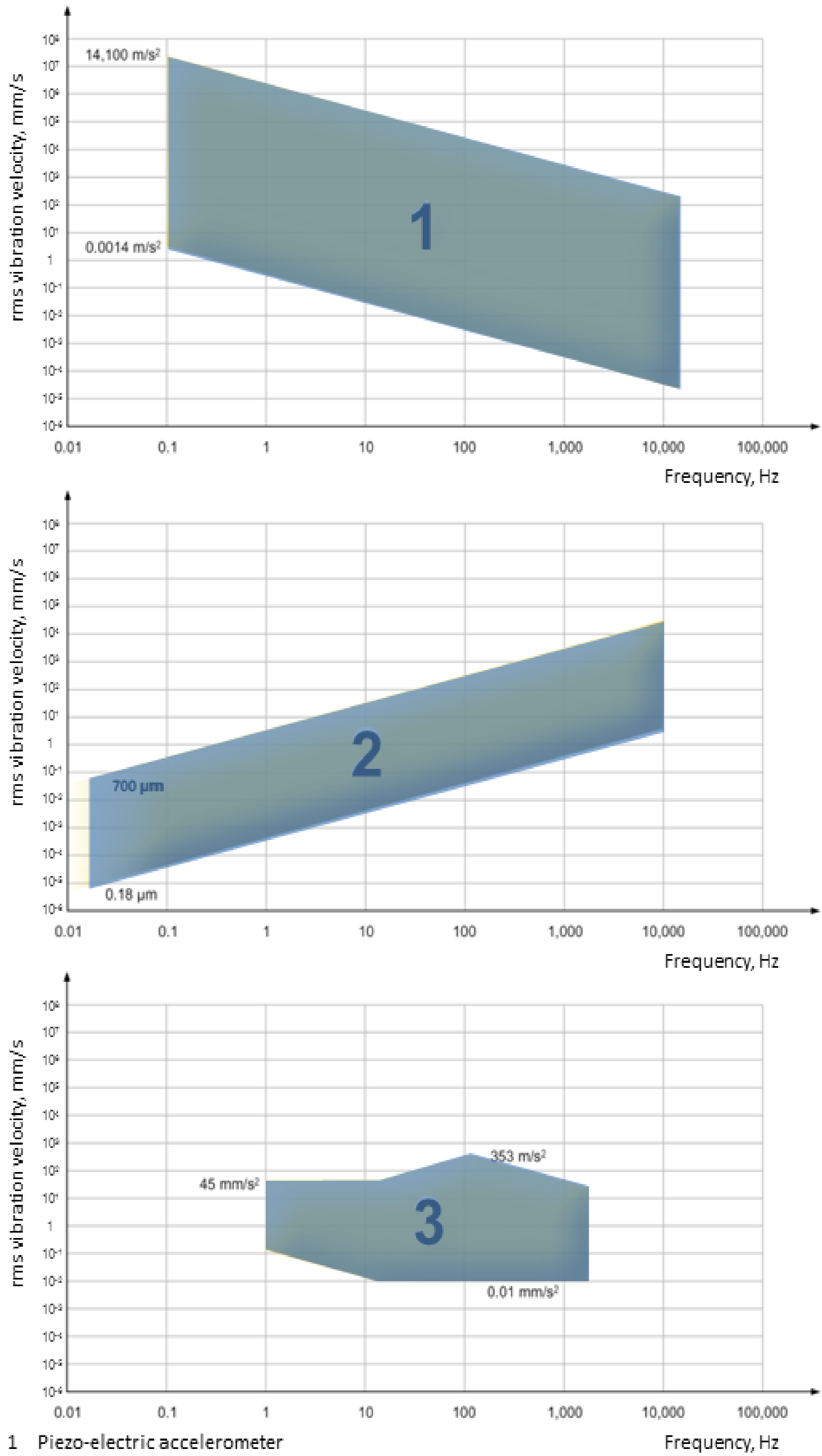
Transducer effective ranges:



1 Piezo-electric accelerometer
2 Eddy-current proximity probe
3 Electro-mechanical velocity transducer

Unit conversions:

Multiply	by	To get
Length, inches (in)	25.4	Millimeters (mm)
Length, Millimeters (mm)	0.0394	Inches (in)
Length, feet (ft)	12	Inches (in)
Weight, Lbf	16	Ounces (oz)
Weight, Ounces (oz)	28.3	Mass, grams (g)
Mass , Kilograms (kg)	2.2	Weight, lbf
Weight, lbf	0.45359	Mass, Kilograms (kg)
Weight lbf	1/386.088	Mass, lbm, lbf s^2 / in
Weight lbf	1/32.174	Mass, lbm, lbf s^2 / ft
Force, lbf	4.448222	Force, Newtons (N)
Mass, Kilograms (kg)	9.81 m/s2	Force, Newtons (N)
Force, Newtons (N)	0.22481	Force, lbf

Notes: "Pounds" (lb) or "weight in pounds" (w) refers to lbf, a unit of force. (ounces (oz)) are also a unit of force. In order to convert to lb or lbf to "mass" (m) or lbm, divide by g, where g is 32.174 ft/s^2 or 386.088 in/s^2. $m=W/g$

Units of mass (lbm) are: lbf s^2/ft or lbf s^2/in

In the metric system Newtons (N) are force and kilograms (or grams) are mass. It is technically incorrect to say something "weighs" 10 kilograms since weight is a unit of force however, this terminology is commonly used.

W=mg = Newtons (N) = kg x 9.81 m/s^2

Vectors:

$$V_{add_x} = A\cos\alpha + B\cos\beta$$

$$V_{add_y} = A\sin\alpha + B\sin\beta$$

$$V_{add} = \sqrt{\left(V_{add_x}\right)^2 + \left(V_{add_y}\right)^2}$$

$$\phi_{add} = \tan^{-1}\frac{V_{add_y}}{V_{add_x}}$$

dB:

$$dB = 20\log\left(\frac{V_m}{V_r}\right)$$

$$\frac{dB_{octave}}{dB_{decade}} = \frac{log(2)}{log(10)} = 0.3$$

$$\frac{V_m}{V_r} = 10^{\frac{dB}{20}}$$

Signal process and data acquisition:

$$T = T_s x N = \frac{N}{F_s} = \frac{N}{2.56 x F_{max}} = \frac{LOR}{F_{max}}$$

T = Time required to collect the waveform
T_s = Time between each sample
N = Number of samples (1024, 2048, 4096, etc.)
F_s = Sampling rate = Samples per second
LOR = Lines Of Resolution (400, 800, 1600, etc.)
F_{max} = Frequency range

$$Separating\ frequency \geq 2\ x\ \frac{F_{max}}{LOR}\ x\ WF$$

$WF = \ Window\ factor$ = 1.0 uniform, 1.5 Hanning, 3.5 flat top window

$$T = \frac{\#\ revs}{Speed} = \frac{\#\ events}{Forcing\ frequency}$$

T = Desired measurement time
$\#\ revs$ = Number of shaft revolutions in time waveform
$Speed$ = Shaft speed
$\#\ events$ = Machine event e.g. tooth mesh
$Forcing\ frequency$ = Frequency of event (e.g. gearmesh frequency)

Unbalance force:

$$U = m.r$$

$$F = m.r.\omega^2 = \frac{w}{g}.r.\omega^2$$

$$F = M.e.\omega^2 = \frac{W}{g}.e.\omega^2$$

$$\omega = 2\pi f = 2\pi\frac{RPM}{60}$$

U = Unbalance (oz-in, gr-in, gr-mm)
F = Force (lbf or N)
m = Mass of balance weight (lbm or kg)
w = Weight of balance weight (lbf or N)
r = Radius of weight (in or m)
ω = Speed of rotation rad/s
f = Frequency Hz
e = Eccentricity of rotor
M = Mass of rotor (lbm or kg)
W = Weight of rotor (lbf or N)
g = 386.1 in/sec^2 or 9.81 m/s^2

$$F_{lbf} = m_{gr}.r_{in}\left(\frac{RPM}{4000}\right)^2$$

$$F_{lbf} = m_{oz}.r_{in}\left(\frac{RPM}{750}\right)^2$$

$$F_{lbf} = m_{lb}.r_{in}\left(\frac{RPM}{188}\right)^2$$

$$F_{kgf} = 0.001\,.m_{gr}.r_{mm}\left(\frac{RPM}{1000}\right)^2$$

Calibration (trial) weights:

$$5\%\,cal.wt. = W_r\left(\frac{168}{RPM}\right)^2$$

$$10\%\,cal.wt. = W_r\left(\frac{238}{RPM}\right)^2$$

$$15\%\,cal.wt. = W_r\left(\frac{291}{RPM}\right)^2$$

W_r = Entire weight of rotor (lb)
cal. wt. = Calibration weight (oz-in)
RPM = Rated speed

Force:

Spring force:

$$F = kx$$

Damping force:

$$F = c\dot{x}$$

Inertia force:

$$F = m\ddot{x}$$

F = Force (lbf or N)
k = Stiffness (lbf/in or N/m)
c = Damping (lbf sec/in or N sec/m)
m = Mass (lbf of kg)
x = Relative deflection (in or m)
$\dot{x}$ = Relative velocity (in/s or m/s)
$\ddot{x}$ = Acceleration (in/sec^2 or m/sec^2)

$$1\ N = 1\ kg\frac{m}{s^2}$$

$$1\ lb_f = 1\ lb_m x\ g = 386.1\ lb_m\frac{in}{s^2}$$

Natural frequency:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k\ g}{W}} = \sqrt{\frac{g}{\Delta}}$$

$$f_n = \frac{1}{2\pi}\omega_n$$

$$\zeta = \frac{C_v}{C_c}$$

$$C_c = 2m\omega_n$$

ω_n= Natural frequency
k = Stiffness (lbf/in or N/m)
m = Mass (lbm or kg)
W = Weight (lbf or N)
Δ = Deflection (in or m)
g = 386.1 in/sec^2 or 9.81 m/s^2
ζ = Damping ratio
C_v = Damping (lbf sec/in or N sec/m)
C_c = Critical damping

$$\omega_n = \sqrt{\frac{K_t}{J}}$$

K_t = Torsional spring stiffness (lbf-in/rad or N-m/rad)
J = Polar inertia (lbf-in-s^2 or N-m-s^2)

Stiffness:

$$k = \frac{W}{\Delta} = \frac{mg}{\Delta}$$

Series: $$\frac{1}{k_T} = \frac{1}{k_S} + \frac{1}{k_S}$$

Parallel: $$k_T = k_P + k_P$$

k_s = Springs in series (lbf/in or N/m)
k_p = Springs in parallel (lbf/in or N/m)
k_T = Total stiffness (lbf/in or N/m)

Unbalance response:

$$X = \frac{\frac{m}{M} e \left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta \frac{\omega}{\omega_n}\right]^2}}$$

X = Rotor response (in or m)
ω_n= Natural frequency
ω = Shaft turning frequency
M = Mass of rotor (lbm or kg)
m = Unbalance mass (lbm or kg)
e = Eccentricity (radius) of mass

$$X = \frac{e \left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta \frac{\omega}{\omega_n}\right]^2}}$$

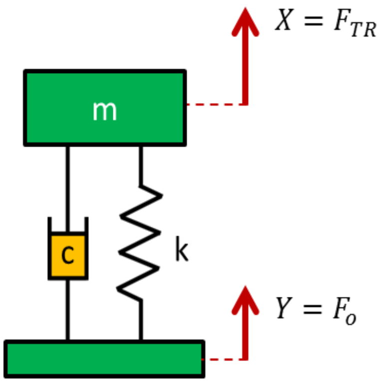
X = Rotor response (in or m)
ω_n= Natural frequency
ω = Shaft turning frequency
e = Eccentricity of rotor (in or m)

$$e = \frac{quality}{W} = \frac{oz\ in}{16W_{lbs}}$$

e = Eccentricity of rotor (in or m)
$quality$ = oz-in, gr-in, gr-mm
W= Weight of rotor
W_{lbs} = Weight of rotor

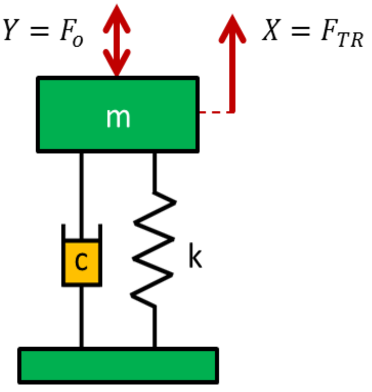
Transmissibility:

$$\frac{X}{Y}=\frac{F_{TR}}{F_o}=\frac{\sqrt{1+\left[2\zeta\frac{\omega}{\omega_n}\right]^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$



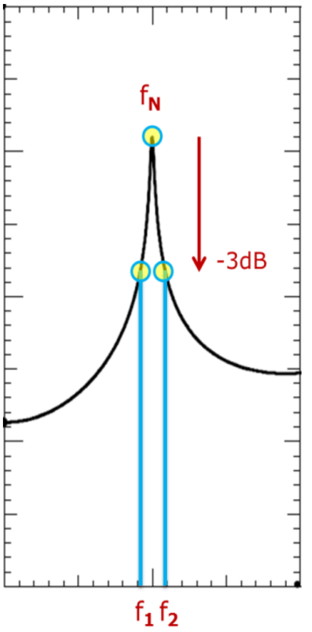
Force response:

$$X=\frac{\frac{F_o}{k}\sqrt{1+\left[2\zeta\frac{\omega}{\omega_n}\right]^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

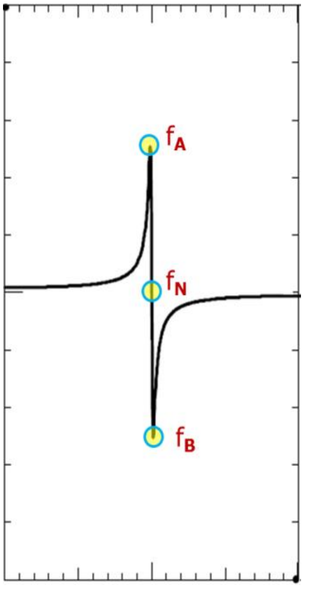


Amplification factor:

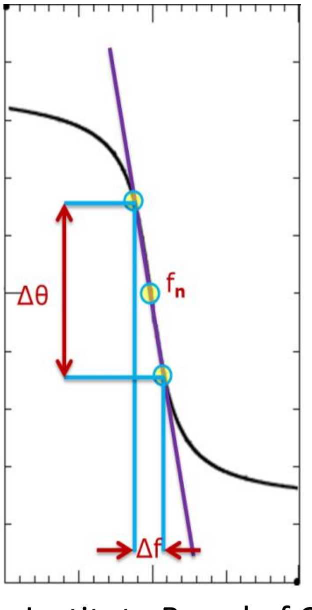
$$Q = \frac{f_N}{f_2 - f_1}$$



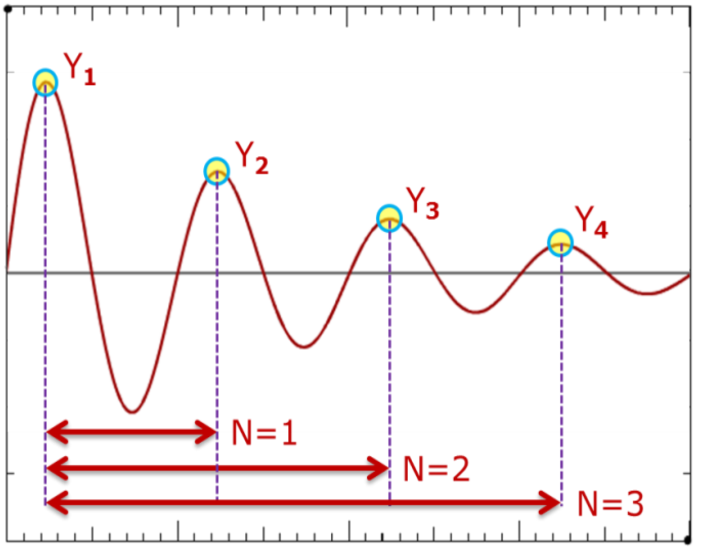
$$Q = \frac{{f_A}^2 + {f_B}^2}{{f_B}^2 - {f_A}^2}$$



$$Q = \frac{\pi\, f_n\, \Delta\theta}{360\, \Delta f}$$



Amplification factor:



$$\delta = \frac{1}{N} ln\left[\frac{Y_1}{Y_{N+1}}\right]$$

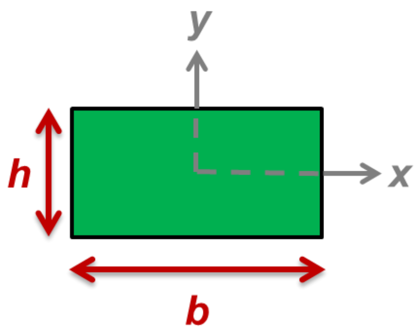
$$\zeta = \frac{1}{\sqrt{1+(\frac{2\pi}{\delta})^2}}$$

$$\zeta = \frac{\delta}{\sqrt{4\pi^2+\delta^2}}$$

$$\zeta = \frac{1}{2\,Q}$$

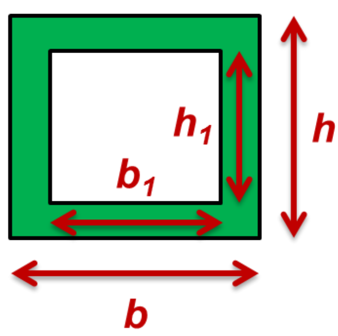
$$\frac{X}{X_o} = Q = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\,\frac{\omega}{\omega_n}\right]^2}}$$

Area moment inertia:



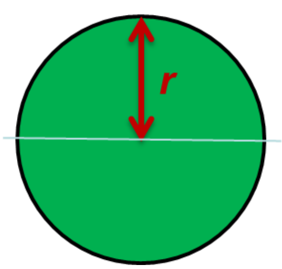
$$I_h = I_x = \frac{bh^3}{12}$$

$$I_b = I_y = \frac{hb^3}{12}$$

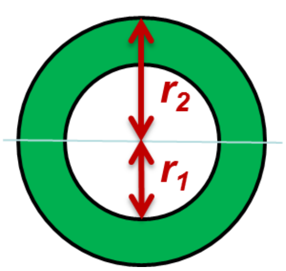


$$I_h = I_x = \frac{bh^3}{12} - \frac{b_1 {h_1}^3}{12}$$

$$I_b = I_y = \frac{hb^3}{12} - \frac{h_1 {b_1}^3}{12}$$



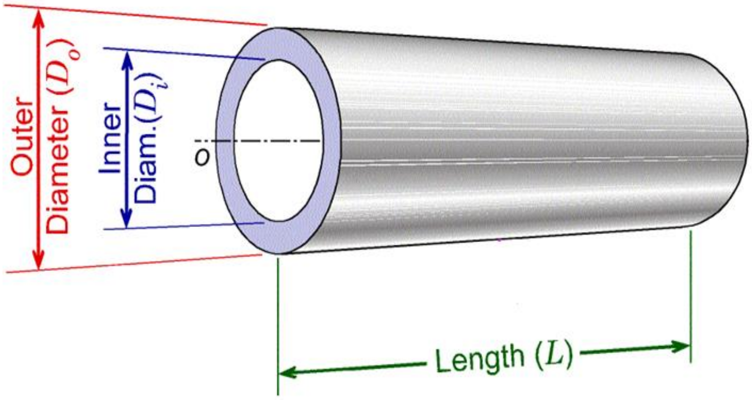
$$I_o = \frac{\pi}{4} r^4$$



$$I_o = \frac{\pi}{4}\left({r_2}^4 - {r_1}^4\right)$$

h = Height (m or in)
b = Width (m or in)
r = Radius (m or in)
I = Moment of inertia (m^4 or in^4)

Polar inertia and stiffness:



$$W = \frac{\pi x L x \rho}{4}\left(D_o{}^2 - D_i{}^2\right)$$

W = Weight (kg)
L = Length (m)
ρ = Density (kg/m^3)
D_o = Outer diameter (m)
D_i = Inner diameter (m)

$$J_p = \frac{\pi x L x \rho}{32 x G}\left(D_o{}^4 - D_i{}^4\right)$$

J_p= Polar inertia (kg-m-s^2)

$$I_t = \frac{J_p}{2} + \frac{\pi x L^3 x \rho}{48 x G}\left(D_o{}^2 - D_i{}^2\right)$$

I_t= Transverse inertia (kg-m-s^2)

$$K_{ax} = \frac{\pi x E}{4 x L}\left(D_o{}^2 - D_i{}^2\right)$$

K_{ax} = Axial stiffness (kg/m)
E = Modulus of elasticity (kg/m^2)

$$K_{rad} = \frac{3 x \pi x E}{4 x L^3}\left(D_o{}^4 - D_i{}^4\right)$$

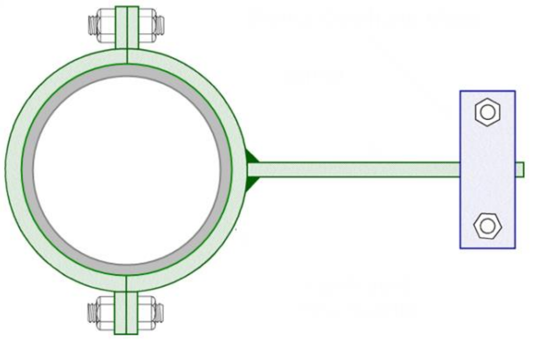
K_{rad} = Radial stiffness (kg/m)

$$K_{tor} = \frac{\pi x G_{shear}}{32 x L}\left(D_o{}^4 - D_i{}^4\right)$$

K_{tor} = Torsional stiff. (kg-m/rad)
G_{shear}= Shear modulus (kg/m^2)

Note: N may be used instead of kg if it is used consistently

Tuned absorber:



$$W_m = \left[\frac{3xGxExI}{L^3 x \omega_c{}^2}\right] - \frac{3}{8} x W_s$$

W_m= Weight of end mass (kg)
W_s= Spring weight (kg)
G = 9.81 m/s^2
L = Length (m)
ρ = Density (kg/m^3)
I = Spring area moment of inertia
E = Modulus of elasticity (kg/m^2)

Lift check multipliers for tilting pad bearings:

Load	3 Pad	4 Pad	5 Pad	6 Pad	7 Pad
LBP	0.667 x Lift	0.707 x Lift	0.894 x Lift	0.866 x Lift	0.948 x Lift
LOP	0.667 x Lift	Lift	0.894 x Lift	Lift	0.948 x Lift

LBP = Load Between Pads
LOP = Load On Pad

$$Bearing\ Diametric\ Clearance = Factor\ x\ Lift$$